647510 **BSc3MatC301x** **Seat No :________**

**B.Sc. Semester - 3 (*CBCS*) Examination**

**December. -2020 [NEW COURSE]**

**Mathematics-M301 (Core)**

**Time: 1:30 Hours** **Marks: 42**

**Instructions:**

1. Figure to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

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**1(A) Answer any two:** **(10)**

(1) Define: Vector Space. If $V = \{(x, y)/x \in R,\ y > 0\}$ and for $(a, b), (c, d) \in V$ and $\alpha \in R$, $(a, b) + (c, d) = (a + c, bd), \alpha(a, b) = (\alpha a, b^{\alpha})$ then show that $V$ is a vector space.

(2) If $V$ is a vector space and $\overline{v_1}, \overline{v_2}, \dots, \overline{v_n} \in V$ such that $Sp\{\overline{v_1}, \overline{v_2}, \dots, \overline{v_n}\} = V$ and $\overline{w_1}, \overline{w_2}, \dots, \overline{w_m}$ are linearly Independent vectors of $V$ then prove that $m \le n$.

(3) Define: Span of a set. Check whether $(1,2,4), (1,5,4), (0,1,2) \in SpA$, where $A = \{(0,1,-1), (0,0,2), (1,3,0)\}$.

**(B) Answer any one:** **(04)**

(1) Define: Sum of two subspaces and direct sum. If $W_1$ and $W_2$ are subspaces of vector space $V$ then prove that $W_1 + W_2$ is also subspace of $V$.

(2) Define: Linearly dependent set. If $A = \left\{sinx, cosx, cos\left(x - \frac{\pi}{4}\right)\right\}$ is a subset of vector space $C[0,2\pi]$ then show that $A$ is linearly dependent set.

**2(A) Answer any two:** **(10)**

(1) Prove that there exists a basis for each finite dimensional vector space.

(2) If $W_1$ and $W_2$ are two subspaces of a finite dimensional vector space V then prove that $dim(W_1 + W_2) = dimW_1 + dimW_2 - dim(W_1 \cap W_2)$

(3) Prove that every linearly independent set of vectors can be extended to form a basis.

**(B) Answer any one:** **(04)**

(1) Define: Basis for vector space. Prove that the set $A = \{1, x, x^2 + x, x^3 + 3x^2 + 2x\}$ is basis for vector space $P_3(R)$.

(2) Extend set $A = \{(1,2,0,0), (1,0,2,1)\}$ of vector space $R^4$ to form a basis for $R^4$.

**3(A) Answer any two:** **(10)**

(1) If $\bar{f}$ and $\bar{g}$ are vector functions defined on the domain D whose first order partial derivatives exist then prove that: $div(\bar{f} \times \bar{g}) = \bar{g} \cdot curl\bar{f} - \bar{f} \cdot curl\bar{g}$

(2) If $\bar{f}$ and $\bar{g}$ are irrotational functions on D then show that $\bar{f} \times \bar{g}$ is a solenoidal function.

(3) If $r = |\bar{r}|$ where $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then prove that: $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$

**(B) Answer any one:** **(04)**

(1) If $\bar{f}$ is solenoidal function then find a, where

$\bar{f} = (ax + 3y + 4z)\hat{\imath} + (x - 2y + 3z)\hat{\jmath} + (3x + 2y - z)\hat{k}$

(2) Define: Gradient of a scalar function. Find the unit normal at the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.

**4(A) Answer any two:** **(10)**

(1) Evaluate: $\iint_R (x^2 + y^2)\, dxdy$ ; where R is the square bounded by the lines $x = y,\ y = -x,\ x - y = 2,\ x + y = 2$.

(2) Evaluate: $\iiint_R (x^2 + y^2 + z^2)^5\, dxdydz$, where R is $x^2 + y^2 + z^2 = a^2$.

(3) Evaluate: $\iiint_R \frac{dxdydz}{(1+x+y+z)^3}$ ; where R is bounded by $x = 0, y = 0, z = 0$ and $x + y + z = 1$.

**(B) Answer any one:** **(04)**

(1) Define: Triple Integral. Find the volume of sphere having radius a using triple integral.

(2) Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xydydx$ and hence evaluate it.

**5(A) Answer any two:** **(10)**

(1) State and prove Green's theorem for a plane.

(2) Verify the divergence theorem for $\bar{V} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and S is a sphere $x^2 + y^2 + z^2 = 1$.

(3) Define: Beta function. Prove that: $\beta(p, q) = \int_0^\infty \frac{y^{p-1}}{(1+y)^{p+q}} dy\ ; p > 0, q > 0$.

**(B) Answer any one:** **(04)**

(1) Prove that the line integral $\int_{(1,1)}^{(x,y)} 2xydx + (x^2 - y^2)dy$ is independent of path and also obtain its value.

(2) Prove that: $\int_0^1 \frac{x^5}{\sqrt{1-x^4}} dx = \frac{\pi}{8}$.

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